

A Study on the Development and Application of Number Theory in Engineering Field

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ABSTRACT

Number theory is probably one of the most important areas of Mathematics used in Computer Science and the basics behind all of modern Cryptography. Number theory is devoted originally to the study of integers. With the contributions made by Mathematicians in different ages to advancing the study of integers, the basics system of number theory has been gradually improved and thereby a complete and unified discipline has been formed. The paper mainly studies the development and application of number theory, aiming to review the history of discipline and explore its influence on production and our life and its applications in Engineering Field. The Number Theory, as such, was less applied in engineering compared to calculus geometry etc. The problem was that it could not be used directly in any application. But the number theory, combined with the computational power of modern computers, gives interesting solutions to real life problem. At present, number theory is widely and fully applied in many fields, such as Computing, Cryptography, Physics, Chemistry, Biology, Acoustics, Electronics, Communication, Graphics and even Musicology.

Keywords: Number theory, Cryptography, Computing, Musicology

Number theory, known as the queen of mathematics is the branch of mathematics that concerns about the positive integers 1, 2, 3, 4, 5 which are often called natural numbers and their appearing properties. From antiquity, these natural numbers classified as odd numbers, even numbers, square numbers, prime numbers, Fibonacci number, triangular number etc.

Due to the dense of unsolved problems, number theory plays a significant role in mathematics.

The recent classification of number theory, depending upon the tools used to address the related problems is shown as:

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Number Theory

Classification	Explanation
Elementary number theory	Elementary number theory is a branch of number theory based on elementary method. In essence, it applies divisible property to mainly study divisible theory and congruence theory. The typical Conclusion in this theory include the familiar Congruence theorem, Euler's theorem, Chinese residual theorem and so on.
Analytic number theory	Analytic number theory studies the integers with calculus and complex analysis. Some analytic functions such as the Riemann function, which studies the properties of integers and primes.
Algebraic number theory	Algebraic number theory is more inclined to study the nature of various rings of integers from the perspective of algebraic structure.
Geometric number theory	Geometric number theory studies the distribution of the integers from the perspective of geometry.
Computation number theory	Computational number theory studies questions in number theory with Computer algorithms.

The research on integers in a scientific way is truly credited to Greeks Latter, a big revolution on this theory happened due to the arrival of the famous book "Elements" by Euclid in which the mathematics itself is depicted with precise proof.

There exit only a few kind of literature discussing on the application of number theory in engineering. So the objective of present work ids to perform a critical review on the existing practices related to the number theory application in engineering field also.

The Development of Number Theory

More than 3000 years ago, the concept of number and arithmetic has occurred. Many questions in number theory have been proposed and then solved, which attracts more and more people to focus on number theory. In the long history, techniques and methods to solve problems have emerged and some theories have been formed. Algebraic number theory has been advanced with the expansion of number field and practical application.

Based of collection of recent data this paper is devote to investigating the birth of algebraic number theory by analyzing the key problems in the development of two higher reciprocity laws and Fermat's theorem.

With a new perspective to observe the history, this paper strives to make more Comprehensive analysis and profound thinking.

1. The stage of Arithmetic

During the period from about 3800 to the 3rd century, arithmetic symbols were not uniform and the algebra was separated from geometry. The ancient Greeks made the greatest Contribution to number theory, including some renowned achievements , such as Euclid's Euclidean algorithm in geometry which proposed that number of prime numbers is infinite and the fundamental theorem of arithmetic which was involved in elementary number theory.

2. The Complete stage of number and equation theory

During the period from the 7th century to the 16th century, irrational and imaginary numbers were discovered.

(a) The discovery of irrational numbers: Hipparchus of the Pythagorean school discovered the first irrational number. He proposed that all numbers could be expressed as the ratio of integers, which led to the first mathematical crisis.

(b) Creations of arithmetic operators and solution to irrational equations: In India, the mathematician Brahmagupta introduced a group of symbols used to express concepts and describe operations in the 7th century and Posgallo later put forward the concept of negative square root, the solution to irrational equations and the algorithm of irrational numbers in the 12th century, which fortered the study of algebra to a new stage.

(c) Establishment of imaginary number theory: In the book “The Great Art” published in 1545 by Milanese Scholar Cardano, the general solution to the cubic equation was unveiled, which was known as Cardano’s formula later. Cardano was the first mathematician to formula the square root of a negative number.

3. The Stage of Linear Algebra

During the period from 17th century to the 19th century, the tools for solving linear problems, matrices, determinants and vector emerged which provideal services to the industrial society.

4. The Stage of Abstract Algebra

During the period from the 19th century to the present, the importance of form and technique to the algebra structure was highlighted, which offered services to the information society.

Application of Number Theory

In the early period, number theory, a branch of pure mathematics was practically less applied in real life. But combined with present Computation technologies, it provides solution to many current problem.

1. Cryptography

Cryptography is one of the essential fields in today’s digital era.

A message sent from sender to receiver in online communication has the risk of being seen by an unknown person without proper safety. This problem is solved by the use of the concept of encryption decryption. The massage which is sent by the sender is said to be ‘encrypted’ or encoded with the help of large number, usually prime, which is said to be a ‘key’, the receiver must have that same key to ‘decrypt’ or decode the message. The application of number theory here is in the generation of such large prime numbers. Maurer^[1] devised an efficient algorithm to generate such numbers with the help of number theory.

When studying number theory, especially cryptography, we pursue deterministic algorithm rather than probabilistic algorithm and we will only lower our requirements and apply probabilistic algorithm if there is no deterministic algorithm.

2. Application Fibonacci Series in Architecture as well as Engineering

Fibonacci sequence, defined by Italian mathematician Leonardo Fibonacci, refers to a series of numbers in which beginning from the third number in the sequence, each number is the sum of the two preceding ones. The n^{th} number in the sequence can be denoted by $f(n) = f(n-1) + f(n-2)$.

The time dependence of moments and size distributions during consolidation is the utilization of Fibonacci Series in the simulation.

Other significant concepts related to the Fibonacci Series.

(a) Golden ratio phi

Any two quantities are said to be in the golden ratio if their ratio is equal to the ratio of their sum to the bigger of the two quantities. Represented algebraically, for two quantities x and y , $x > y > 0$,

$$(x + y) / x = x/y = \text{phi.}$$

Shapes of several natural and human made objects are seen to obey the Golden Ratio^[2]. The Spirals in the flowers of plants and Parthenon, the famous monument are same classical examples. Fibonacci series has found many uses in architecture as well as engineering and is widely seen in nature. The phi code explains the behavior of structural elements used in engineering. It is seen as a defining parameter in the stress analysis of beams. Collins and Brebbia^[3] pointed out the existence of phi code in the relation between normal and shear stresses.

(b) Pascal Triangle

The numbers on diagonals of Pascal triangle add to the Fibonacci Sequence.

(c) Area of a rectangle

The sequence of the first few numbers in the Fibonacci Sequence are treated as different small quadrilateral areas, and they can be combined into large quadrilateral areas.

3. Computer Animation

Linear transformation is usually used to make images and computer graphics are to build graphic on display devices through algorithms and programs, so linear transformation technology can be used to make Computer animation. Computer graphics mainly consist of image representation, storage and computation. With the improvement of softer capabilities, linear transformation technology is commonly used in Computer animation.

4. Machine Translation

The main algorithm of Machine translation is based on the Statistical method, with the accuracy of 90%. In addition, this algorithm is used in image search technology. The core of this method is that the language

units of source language and target language can be represented by vectors and the lexical vectors of different languages can be projected into a two-dimensional plan for analysis. Experimental results show that the lexical vectors of different languages do have some relations similar to linear relations, so it is of significance to classify machine translation as a linear transformation.

5. Others Fields

Number theory also plays a surprising role in other theories. In quantum theory, Hermite operator is one of the most basics concept. One of the famous theorems in mathematics is the Pythagoras theorem. It deals with the right-angled triangles, giving the relations between the sides. Unsurprisingly, it has applications in any field which deals with triangles. Few famous examples are the wing configuration used in modern jet aircraft in the 'Delta Wing'. The theorem plays apart in the effective and efficient design of such configuration . Similar application can be found in tios of rockets, which is an isosceles triangle in sectional view. Sectional analysis of frustum of cones which serves as fairing between the stages of a multi-stage rocket, is also an example.

Calculations of propeller and engine blade angles involve the theorem. Aerospace Scientists and meteorologists find a range and sound source using this theorem.

The acoustic quality of Concert halls is be improved with the help of number theory as discussed by Manfred^[4]. The Construction of new musical scales to the optimum diffusion of sound in the halls improves the acoustic quality to a great practical extent.

New polynomial equivalents of Jacobi's triple product were given by Krishnaswami Alladi and Alexander Berkovich^[5].

Apart from that, number theory is also widely used in non-mathematical disciplines, such as information science, theoretical physics, quantum chemistry and so on.

A introduction to both the mathematical and engineering forms of Coding theory, Weights of Code words were discussed by Robert and Howard. Application of Ramanujan sum in single processing was noticed over the past decades. Vaidyanathan^[6] showed the details of Ramanujan Subspace and studied its prospects for improving the decomposition of finite duration signals into the finite sum of orthogonal substances.

Thus the extensive applications of number theory noted in several areas. The current scenario is such that the role of number theory has more weight for cyber security problems.

CONCLUSION

This paper mainly discussed the basic concept, theory, development process and applications of number theory. Various engineering applications of number theory were mentioned in detail. The significant contribution of number theory in recent years is in the area of cryptography and hence Computer Science engineering was noted.

The versatility of applications was also recognized. By reviewing the development of number theory and its application, this paper's aims to help reader acquire the origin and development of number theory and its future trend in the Combination of Computer Science. In today's society, with the rapid development of Computer field, number theory or even mathematical discipline will make greater strides in the future.

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